NUCLEON AND PION FORM FACTORS IN DIFFERENT FORMS OF RELATIVISTIC KINEMATICS

B. DESPLANQUES

Laboratoire de Physique Subatomique et de Cosmologie (UMR IN2P3/CNRS-UJF-INPG),
F-38026 Grenoble Cedex, France

Calculations of form factors in different forms of relativistic kinematics are presented. They involve the instant, front and point forms. In the two first cases, different kinematical conditions are considered while in the latter case, both a Dirac-inspired approach and a hyperplane-based one are incorporated in our study. Numerical results are presented for the pion form factors with emphasis on both the low and high Q^2 range. A new argument is presented, explaining why some approaches do considerably much better than other ones when only a single-particle current is considered.

Keywords: 03.65.Pm; 21.45.+v; 13.40.Gp

1. Introduction

The knowledge of hadron form factors, especially for the nucleon and the pion ones, represents an important source of information about the structure of the systems under consideration. By varying the momentum transfer, large as well as small distances can be explored, allowing one to learn about hadronic physics in the perturbative and non-perturbative regimes of QCD and its modelization. Involving large momentum transfers, the above study supposes that a reliable implementation of relativity is made. This is mandatory if some information about the hadronic structure is to be looked for from experiments.

There are many ways to implement relativity in the description of properties of a few-body system. The most ambitious one is based on field theory but, at present, its use for the nucleon form factor is hardly conceivable. A quite different approach involves relativistic quantum mechanics (RQM), which contrary to the previous one, assumes a fixed number of degrees of freedom. Less fundamental, this one is however more adapted when a modelization of hadrons from constituent particles is used, as most often done. Following Dirac,[1] many approaches along these lines have been proposed depending on the symmetry properties of the hypersurface on which physics is described. This reflects in the construction of the Poincaré group generators, which drop accordingly into dynamical and kinematical operators.

When calculating properties such as form factors, all approaches should converge to a unique answer but, of course, some may be more convenient in that the bulk contribution is produced by a one-body current. In other ones, large contributions

2   *B. Desplanques*

from two- or many body-currents may be required, possibly obscuring conclusions about the physics. This requires that independent studies be performed to establish the respective advantage of various approaches by comparing their predictions.

Those studies that will be presented here have largely been motivated by the successful description of the nucleon form factors in the "point-form" approach[2] while a standard front-form one[3] is failing in the same conditions. Adding to this puzzling situation, it is noticed that accounting for the well known physics underlying the vector-meson-dominance phenomenology, ignored in the former case, would reduce the discrepancy in the latter one. For the present purpose however, we will consider a system simpler than the nucleon, namely the pion. Apart from the fact that there is an evident logics in considering systems with increasing complexity, the smallness of the pion mass in comparison with the sum of the constituent masses turns out to considerably enhance the differences between various approaches. This can contribute to sharpen the conclusions.

The plan of the paper is as follows. In the second section, we precise the ingredients entering the calculation of form factors in different kinematics of relativitic quantum mechanics (RQM): instant, front and point forms. For each approach different cases, described in the text, are considered. Results for both the charge and scalar pion form factors are presented and discussed in the third section. Some attention is given to their asymptotic behavior. The fourth section is devoted to the conclusion where the role of the space-time translation invariance is evoked. Due to limited space, we skip many details and refer to published works for them.[4,5,6]

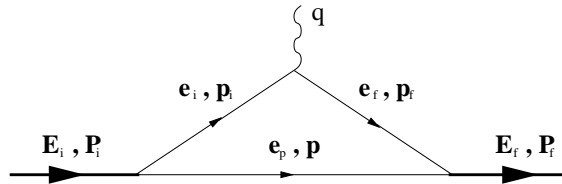## 2. Different forms of relativistic quantum mechanics: a few points

Fig. 1.   Contribution to the form factor in the single-particle approximation

In order to calculate form factors of a given system, two ingredients are needed: the relation between the momenta of its constituents and the total momentum, which characterizes each RQM approach (see kinematics in Fig. 1), and a solution of a mass operator, which can be chosen as independent of the approach. They are successively discussed in the following.

For the two-body system of interest here, the relation between the momenta of its constituents and the total momentum takes a unique form. This one reads:

$$\vec{p}_1 + \vec{p}_2 - \vec{P} = \frac{\vec{\xi}}{\xi^0}\,(e_1 + e_2 - E_P), \tag{1}$$

where the 4-vector, ξ^μ, is representative of the symmetry properties (if any) evidenced by the hypersurface which physics is described on. Accordingly, the Poincaré group generators, P^μ (P^0, $\vec{P}$) and $M^{\mu\nu}$ ($\vec{K}$, $\vec{J}$) drop into dynamical or kinematical ones.[7] This character together with the 4-vector ξ^μ are precised below:

- instant form: $t = \tau$, $\quad \xi^0 = 1$, $\quad \vec{\xi} = 0$;

     dynamical: P^0, $\vec{K}$,   kinematical: $\vec{P}$, $\vec{J}$,

- front form: $t - \vec{n} \cdot \vec{x} = \tau$, $\quad \xi^0 = 1$, $\quad \vec{\xi} = \vec{n}$,

    where $\vec{n}$ is a unit vector with a fixed direction, generally chosen opposite to the z-axis orientation ($\xi^2 = 0$);

     dynamical: $P^0 - P^z$, $J_\perp$,   kinematical: $P^0 + P^z$, $P_\perp$, J^z, K^z, $K_\perp - \hat{z} \times J_\perp$,

- Dirac's inspired point form: $t^2 - \vec{x}^2 = \tau$, $\quad \xi^0 = u^0 = 1$, $\quad \vec{\xi} = \vec{u}$,[5]

    where $\vec{u}$ is a unit vector that points to any direction, consistently with the absence of a particular 3-direction on a hyperboloid ($\xi^2 = 0$);

     dynamical: P^0, $\vec{P}$,   kinematical: $\vec{K}$, $\vec{J}$.

An "instant-form" approach "which displays the symmetry properties inherently present in the point-form" one has been proposed.[8] The Poincaré group generators, P^μ and $M^{\mu\nu}$, have respectively a dynamical and a kinematical character, as for the Dirac's point form. However, as noticed by Sokolov,[9] it implies physics described on an hyperplane perpendicular to the velocity of the system under consideration (hypersurface $v \cdot x = \tau$). It therefore differs from the Dirac's one. This "point form", which has been referred to in many recent applications,[2,10,11,12,13] evidences specific features. Contrary to the other approaches mentioned above, the 4-vector, ξ^μ, depends on the properties of the system ($\xi^\mu \propto P^\mu$). This approach is also on a different footing with other respects.[5]

For the mass operator, we refer to an equation used in our previous works[4,5,6] with appropriate changes due to the 1/2-spin of the constituents.[13] For our purpose, which is mainly to compare different approaches between themselves rather than to experiment, we include in the interaction a confining potential with string tension, $\sigma_{s.t.} = 1 \,\text{GeV/fm}$ and a gluon exchange one with strength $\alpha_s = 0.35$. This last contribution is of relevance to test the ability of RQM approaches in reproducing the expected QCD asymptotic behavior of form factors. As been noticed,[14] this behavior is closely related to the most singular part of the interaction at short distances. The quark and pion masses are taken as $m_q = 0.3 \,\text{GeV}$ and $M_\pi = 0.14 \,\text{GeV}$.

Expressions of the single-particle contribution to form factors in the spinless case have been given elsewhere (see for instance Ref. 6). They can be expressed as an integral over the spectator-particle momentum. Interestingly, they take a unique form in most cases, which allows one to discard major biases in the comparison of different approaches. Their derivation supposes to express the momenta of the constituents, $\vec{p}_{i,f}$ and $\vec{p}$ in Fig. 1 in terms of the total momentum, $\vec{P}$, the internal variable appearing in the mass operator, $\vec{k}$, and the 4-vector, ξ^μ. The relation of $\vec{p}$'s to the $\vec{k}$ variable assumes a Lorentz-type transformation while fulfilling Eq. (1). It

4   *B. Desplanques*

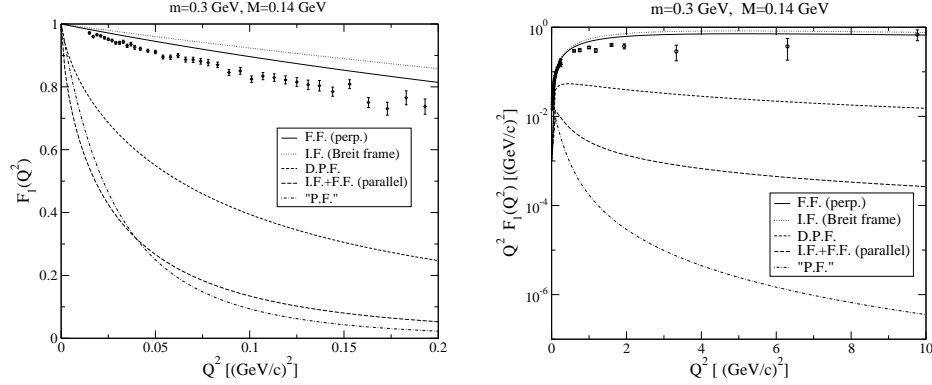is nothing but the Bakamjian-Thomas one in a particular case.[15]

Fig. 2.   Pion charge form factor at low and high Q^2 together with experimental data

3. Results for the charge and scalar pion form factors

The pion has two form factors: the charge one, $F_1(Q^2)$, for which measurements
are available[16,17,18] and a scalar one, $F_0(Q^2)$, which, in absence of an appropri-
ate probe, is unknown but can be nevertheless useful for a comparison of different
approaches. The low and high Q^2 behaviors of $F_1(Q^2)$, in relation with the charge
radius or the asymptotic behavior, are of special interest. Moreover, as the instant-
and front-form form factors are not Lorentz invariant, they can be considered for var-
ious kinematical configurations. Besides the standard ones (respectively Breit frame
and $q^+ = 0$), we consider both of them for a parallel kinematics and $|\vec{P}_i + \vec{P}_f| \to \infty$,
where they coincide. We also consider results in two point-form approaches, which
contrary to the other forms, are Lorentz invariant.

Results for $F_1(Q^2)$ are presented in Fig. 2. They clearly fall into two sets: the
standard instant- and front-form form factors that are relatively close to experiment
and the other ones that are far apart. Looking in detail at these last ones, it is
found that they roughly depend on the momentum transfer Q through the quantity
$(2\bar{e}_k/M_\pi)\,Q$, hence a charge radius scaling like the inverse of the pion mass, which
explains the rapid fall off of the corresponding form factors at low Q^2 (a rapid fall off
is also found in truncated field-theory calculations[19,20,21]). At higher Q^2, it sounds
that the Q^{-2} asymptotic behavior is reached. Actually, examination of the charge
form factor at much higher Q^2 indicates that the behavior is Q^{-4} (see Fig. 3). The
consideration of the scalar form factor, $F_0(Q^2)$, is especially useful here. As Fig. 3
shows, this form factor has the QCD-expected Q^{-2} asymptotic behavior, indicating
that there is nothing wrong with the solution of the mass operator we used. In
order to get the right power-law behavior for the charge form factor, we considered
the contribution of two-body currents (pair-type term). This one, determined so

that to reproduce the full Born amplitude,[22] has been calculated in the instant form. As Fig. 3 shows, it provides the right Q^{-2} asymptotic behavior. Moreover, the coefficient has the expected expression (up to a numerical factor).
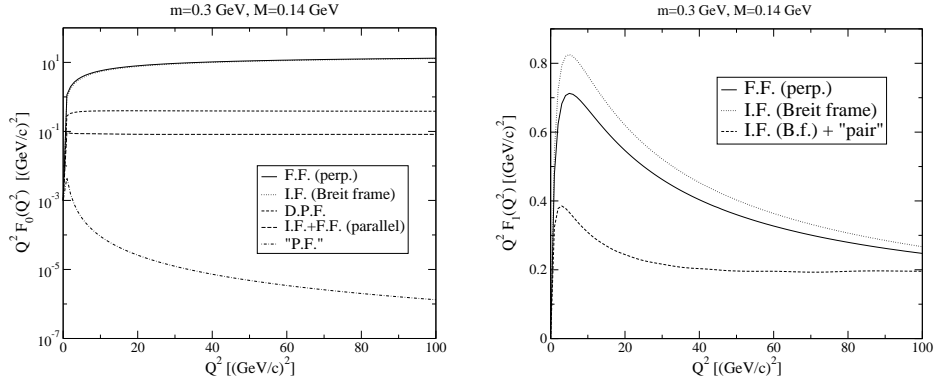

Fig. 3.   Pion scalar and charge form factor in the asymptotic regime

4. Conclusion and prospect

The examination of the pion charge form factor calculated from a single-particle current in different RQM approaches shows unambiguously that results fall into "good" and "bad" ones. The conclusion is not to be affected by refining the physical description as the discrepancy in the last case reaches huge factors. It fully confirms the conclusions achieved in the spinless case whose physical description is simpler.[6] The QCD asymptotic behavior is obtained from two-body currents.

Lorentz invariance is often advertised as a validity criterion of some approach. This view is not however supported by present point-form results, which explicitly evidence the above invariance property. Moreover, the violation of Lorentz invariance, as measured from the rather small discrepancy between the standard instant- and front-form results, does not seem to be necessarily large. Another criterion has therefore to be found. In a field-theory approach, the 4-momentum is conserved at the vertex representing the interaction of constituents with the external probe. This cannot be generally fulfilled in RQM approaches at the operator level (unless many-body currents are considered). One can however require that the property be verified at the level of the matrix element. Considering this weaker argument, it allows one to account for the observed discrimination of results into "good" and "bad" ones. As the 4-momentum conservation stems from Poincaré space-time translation invariance, fulfilling this property could be the relevant criterion. The above invariance may also be the important symmetry whose violation is suggested by the peculiar behavior of some form factors in the limit of a zero-mass system.[6,22]

6   *B. Desplanques*

Poincaré space-time translation invariance implies relations such as:[23]

$$\left[P^\mu \ , \ J^\nu(x) \right] = -i \, \partial^\mu \, J^\nu(x), \qquad (2)$$

which can be used for a quantitative check. Considering a single-particle current, it is found that the equality is satisfied at the matrix-element level for the standard instant- and front-form results. It is violated in all the other cases. Skipping details, one finds that the l.h.s. and r.h.s respectively involve the quantities Q and $(2\bar{e_k}/M_\pi) \, Q$. The extra factor at the r.h.s., $(2\bar{e_k}/M_\pi)$, which is the same as the one explaining the discrepancy between the "good" and "bad" form factors in Fig. 2, provides a measure of the violation of Poincaré space-time translation invariance. To get rid of it, interaction currents should be considered. Schematically, their effect could combine with the kinetic energy term, $2\,\bar{e_k}$, at the numerator of the above factor so that the overall factor be 1 (using $2\,\bar{e_k} + V = M_\pi$).[22]

Coming back to the motivation of the present work, it is noticed that the success of the point-form description of the nucleon form factors is mostly due to a factor similar to the above one. As a more complete calculation is expected to remove this factor, a situation similar to the standard front-form calculation should be recovered. We believe it is doubly fortunate. The description of the nucleon form factor could now incorporate the well known vector-meson dominance phenomenology. The difficulty to reconcile the point-form descriptions of the nucleon and pion form factors, respectively good[2] and bad,[13] vanishes.

While Lorentz invariance has often been advocated in calculating form factors, Poincaré space-time translation invariance could be a more relevant property.

Acknowledgements

References

1. P.A.M. Dirac, *Rev. Mod. Phys.* **21**, 392 (1949).
2. R.F. Wagenbrunn, *et al.*, *Phys. Lett.* **B511**, 33 (2001).
3. F. Cardarelli, E. Pace, G. Salmè, S. Simula, *Phys. Lett.* **B357**, 267 (1995).
4. A. Amghar, B. Desplanques, L. Theußl, *Nucl. Phys.* **A714**, 213 (2003).
5. B. Desplanques, nucl-th/0405059, *Nucl. Phys.* (accepted).
6. B. Desplanques, preprint, nucl-th/0407074.
7. B.D. Keister and W. Polyzou, *Adv. Nucl. Phys.* **20**, 225 (1991)
8. B. Bakamjian, *Phys. Rev.* **121**, 1849 (1961).
9. S.N. Sokolov, *Theor. Math. Phys.* **62**, 140 (1985).
10. T.W. Allen, W.H. Klink, *Phys. Rev.* **C58**, 3670 (1998).
11. T.W. Allen, W.H. Klink, W.N. Polyzou, *Phys. Rev.* **C63**, 034002 (2001).
12. B. Desplanques, L. Theußl, *Eur. Phys. J.* **A13**, 461 (2002).
13. A. Amghar, B. Desplanques, L. Theußl, *Phys. Lett.* **B574**, 201 (2003).
14. C. Alabiso, G. Schierholz, *Phys. Rev.* **D10**, 960 (1974).

15. B. Bakamjian, L.H. Thomas, *Phys. Rev.* **92**, 1300 (1953).
16. C.J. Bebek, *et al.*, *Phys. Rev.* **D17**, 1693 (1978).
17. S.R. Amendolia, *et al.*, *Nucl. Phys.* **B277**, 168 (1986).
18. J. Volmer, *et al.*, *Phys. Rev. Lett.* **86**, 1713 (2001).
19. B.L.G. Bakker, H.-M. Choi, C.-R. Ji, *Phys. Rev.* **D63**, 074014 (2001).
20. S. Simula, *Phys. Rev.* **C66**, 035201 (2002).
21. J.P.B.C. de Melo, *et al.*, *Nucl. Phys.* **A707**, 399 (2002).
22. B. Desplanques, L. Theußl, *Eur. Phys. J.* **A21**, 93 (2004).
23. F.M. Lev, *Rivista del Nuovo Cimento* **16**, 1 (1993).